

The Origin of Anomalous Low-Temperature Downturns in the Thermal Conductivity of Cuprates

M. F. Smith[†], Johnpierre Paglione^{††}, and M. B. Walker[‡]
Department of Physics, University of Toronto, Toronto, Canada.

Louis Taillefer[‡]

Département de physique et Regroupement québécois sur les matériaux de pointe, Université de Sherbrooke, Sherbrooke, Canada
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We show that the anomalous decrease in the thermal conductivity of cuprates below 300 mK, as has been observed recently in several cuprate materials including $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_{7-\delta}$ in the field-induced normal state, is due to the thermal decoupling of phonons and electrons in the sample. Upon lowering the temperature, the phonon-electron heat transfer rate decreases and, as a result, a heat current bottleneck develops between the phonons, which can in some cases be primarily responsible for heating the sample, and the electrons. The contribution that the electrons make to the total low- T heat current is thus limited by the phonon-electron heat transfer rate, and falls rapidly with decreasing temperature, resulting in the apparent low- T downturn of the thermal conductivity. We obtain the temperature and magnetic field dependence of the low- T thermal conductivity in the presence of phonon-electron thermal decoupling and find good agreement with the data in both the normal and superconducting states.

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I. INTRODUCTION

In recent measurements made of the thermal conductivity κ of high T_C cuprates at milliKelvin temperatures, an unexplained decrease in the electronic component of thermal conductivity with decreasing temperature well below 1 K has been observed (we will refer to such decreases as “downturns”). The most dramatic observation of a downturn in κ/T occurred in the field induced normal state of optimally doped $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_{7-\delta}$ (henceforth PCCO), where the electronic contribution κ_{el}/T was found to decrease below 300 mK and vanish at the lowest measured temperatures¹. This was in startling contrast to electrical conductivity measurements made on the same material (and to NMR studies²), which showed apparently normal metallic behaviour over the same temperature range. These measurements appeared to show an unexpected violation of the Wiedemann-Franz law at very low temperatures¹, with heat conductivity becoming much less than charge conductivity. (Note that the main effect, observed above 300 mK, is a violation whereby heat conductivity is greater than charge conductivity by a factor of 1.7-1.8.)¹

Downturns have been observed in other cuprate materials in both the normal and superconducting states. For example, data on non-superconducting, overdoped $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and on optimally and overdoped samples of superconducting $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ and $\text{YBa}_2\text{Cu}_3\text{O}_{7-\delta}$ (henceforth LSCO and YBCO, respectively) have showed similar downturns^{3,4,5,6}. In the d-wave superconducting state the downturn is associated with a decrease in the nodal quasiparticle contribution to κ/T at a temperature well below that corresponding to the impurity scattering rate. This cannot be understood within standard dirty d-wave transport theory⁷.

In this article we show how the observed downturns can be attributed to a decrease in the rate of energy transferred from the phonons that provide the heat current to the electrons being studied (from now on we will use the word “electron” to refer to either a Landau quasiparticle in the Fermi liquid or a nodal Bogolubov quasiparticle in the d-wave superconductor). As a result of the reduced heat transfer rate, the phonons no longer come into thermal equilibrium with the electrons at very low temperatures and the electronic heat current becomes dependent on the phonon-electron heat transfer rate, and thus decreases rapidly with temperature. With the origin of the downturn thus understood, the low- T thermal conductivity data for cuprates in the superconducting state are consistent with the predictions of dirty-d-wave theory. Similarly, the thermal conductivity of normal state PCCO at very low temperatures exhibits a constant linear term in κ/T versus T down to the lowest temperatures, consistent with the behaviour measured above 300 mK, with no downturn. A detailed discussion of the experimental results in a variety of materials will appear in Ref. 8.

We develop a theoretical model of the temperature and field dependence of the downturn by calculating the phonon-electron heat transfer rate and using it to determine the measured heat current and the thermal conductivity. In summary, our results (i) give the correct temperature dependence for the observed thermal conductivity (ii) give the correct order of magnitude (which depends on the electron-phonon matrix element) of the observed effect (iii) give the correct dependence of the thermal conductivity on magnetic field in the vortex state.

In Section II of this article we describe the role that phonon-electron heat transfer plays in low-temperature thermal conductivity measurements and explain the ori-

gin of the downturn. In Section III we show the calculation of the relevant electron-phonon heat transfer rate in the normal state and in the vortex state of a d-wave superconductor as a function of temperature and magnetic field. In Section IV we will compare our results with data. In Section V we discuss some implications of our results.

II. THE EFFECT OF PHONON-ELECTRON HEAT TRANSFER ON THE OBSERVED THERMAL CONDUCTIVITY AT LOW TEMPERATURE

In order to study the effect of phonon-electron heat transfer rate on the low- T thermal conductivity data, we use a simplified model (Fig. 1) of the experimental configuration. A thermal current, Q , is carried into the sample by phonons and electrons. The “contact” resistors, labelled $R_{el(c)}$ and $R_{ph(c)}$, correspond to the thermal resistance encountered by the electronic and phononic heat currents before entering the sample. Within the cuprate material, the thermal resistance to the electron heat current, R_{el} , and to the phonon heat current R_{ph} are assumed to be described by the standard theory, i.e. by Fermi liquid transport theory for the normal state and dirty-d-wave transport theory for the superconducting state. The temperature and field dependent phonon-electron thermal resistance R_{el-ph} , describes the heat transfer between phonons and electrons in the sample.

The model of Fig. 1 is clearly a crude approximation of the actual experimental configuration, but it turns out to be sufficient to describe the effect of phonon-electron heat transfer on the low- T thermal conductivity measurements. We will here briefly mention some of the differences between the model shown and the actual experiment. According to Fig. 1, the temperature difference between phonons and electrons exists only at the hot end of the sample and the transfer of heat between phonons and electrons occurs before either the phonon or electron heat currents encounter any resistance from the sample. In reality, a temperature difference between phonons and electrons can be present everywhere in the sample and thus heat transfer can occur throughout the sample volume. The temperature difference that drives the heat current through R_{el-ph} actually depends on position within the sample. However, since all our results below are calculated to linear order in the temperature difference between electrons and phonons, we can determine the total phonon-electron heat transfer rate below as if a constant temperature difference existed throughout the sample volume. Also, in Fig. 1 we have shown only one set of current contacts, at the hot end of the sample, although a similar set of contacts must be present at the cold end of sample in the experimental configuration. One could slightly improve the model of Fig. 1 by including a second, equivalent, set of resistors: $R_{el(c)}$, $R_{ph(c)}$ and R_{el-ph} at the cold end of the sample (the right

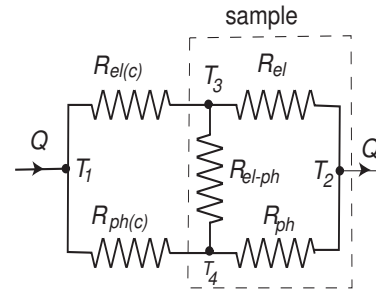


FIG. 1: A simple picture of the experimental configuration for thermal conductivity measurements. The thermal resistance to phonon and electron heat currents that occurs before the current enters the sample are represented by $R_{ph(c)}$ and $R_{el(c)}$, respectively. The thermal resistance to phonon and electron heat flow through the sample are represented by R_{ph} and R_{el} , respectively. The electron-phonon heat transfer rate is associated with R_{el-ph} . Temperatures at different positions have been labelled.

side of the figure). This would account for effects of the thermal resistance encountered by the phonon and electron heat currents after leaving the sample and would allow for a temperature difference and resulting heat transfer between electrons and phonons to occur at the cold end of the sample as well as the hot end. We have determined the thermal conductivity for this modified model and have found that the effect of the modification to our main result below (Eq. 5) is the same as that of simply doubling both the value of $R_{el(c)}$ and the effective sample length over which phonon-electron heat transfer occurs. Since we can only make rough comparisons of $R_{el(c)}$ (as we discuss in Section IV) and the sample length (see footnote, Ref. 9) with measured quantities, this modification does not significantly change any of our results below. We include these factors of two in comparisons with data below but will avoid needlessly cluttering the figure shown by including additional resistors.

As indicated in the introduction, the electron-phonon thermal resistance R_{el-ph} plays a central role in the determination of the low- T behaviour of the apparent electronic contribution to the thermal conductivity. In Section III, we calculate the electron-phonon thermal resistance for both the normal metallic state and the d-wave superconducting state and find that the result in each case is of the following form:

$$R_{el-ph}^{-1} = \Omega K T^n \quad (1)$$

where Ω is the sample volume, K is a constant, which depends on the electron-phonon matrix element, that we determine and n is between 4 and 5. For the remainder of this section, we will use Eq. 1 along with Fig. 1 in order to determine the low- T behaviour of the apparent electronic thermal conductivity. This is done to provide

a general explanation for the observed low- T downturns in both the normal state and the d-wave superconducting state before getting into the details of the calculation of R_{el-ph} for these separate cases.

According to Eq. 1, R_{el-ph} varies more rapidly with temperature than any of the other resistors in Fig. 1. At sufficiently high temperatures, R_{el-ph} is negligible and the phonons and electrons at the hot end of the sample are in thermal equilibrium, so $T_3 = T_4$. The thermal conductivity κ can then be obtained from the thermal conductance G , where $G = Q/(T_4 - T_2) = R_{el}^{-1} + R_{ph}^{-1}$. The result for either the Fermi liquid or the dirty-d-wave superconductor using the scattering rates dominant at low- T then has the usual form

$$\kappa/T = \alpha + \beta T^2 \quad (T \gg T_D) \quad (2)$$

where the αT term is associated with the electron heat current and the βT^3 term is associated with the phonon heat current. We have introduced the characteristic downturn temperature T_D , which corresponds to the temperature at which $R_{el-ph} = R_{el}$, that is the relevant temperature scale for the downturn in the electronic contribution to κ .

At very low temperatures, R_{el-ph} becomes large, and the phonons and electrons in the sample become thermally decoupled. There arises a temperature difference between the electrons and the phonons at the hot end of the sample, $T_3 \neq T_4$, that depends on the relative magnitude of $R_{el(c)}$ and $R_{ph(c)}$. There is thus some ambiguity in the definition of the thermal conductivity. A thermometer at the hot end of the sample could measure T_3 , T_4 , or some function of the two, say $T' = f(T_3, T_4)$ depending on its relative sensitivity to phonons and to electrons. The thermal conductivity at low-temperature would be determined from a conductance given by $G = Q/(T' - T_2)$. One could consider many possible cases¹⁰ resulting from different forms of $f(T_3, T_4)$ over the relevant temperature range. Here we focus on one limiting case, $f(T_3, T_4) = T_4$ corresponding to a thermometer sensitive only to phonon properties. In this limiting case, an exact analysis of the configuration of Fig. 1 gives the conductance

$$G \equiv \frac{Q}{T_4 - T_2} = \frac{1}{R_{el}} \left(\frac{1 + \frac{R_{ph(c)}}{R_{ph}} \Lambda}{1 + \frac{R_{el(c)}}{R_{el}} \Lambda} \right) + \frac{1}{R_{ph}} \quad (3)$$

where

$$\Lambda = \frac{R_{el-ph}}{R_{el-ph} + R_{el(c)} + R_{ph(c)}}.$$

In this article we do not try to understand the nature of the contact resistances $R_{el(c)}$ and $R_{ph(c)}$ by looking into the details of the experiments, but instead determine what relative values these quantities must have in order for us to be able to account for the observed low-temperature thermal conductivity downturns. To this

end, note that when $R_{el(c)}$ becomes infinite, one finds the thermal conductance

$$\frac{Q}{T_4 - T_2} = \frac{1}{R_{el} + R_{el-ph}} + \frac{1}{R_{ph}} \quad (4)$$

where again we have assumed that the hot end of the sample measures the phonon temperature T_4 . At $T \gg T_D$ where $R_{el-ph} \ll R_{el}$, Eq. 4 gives Eq. 2 above. However, at temperatures $T \ll T_D$, where R_{el-ph} describes a high resistance to heat flow between phonons and electrons, the effective electronic contribution to the heat conductivity (the first term on the right side of Eq. 4) is much reduced. This is our picture of the low-temperature downturns. Alternatively, if the value of $R_{ph(c)}$ is taken to be infinite, the determined thermal conductivity shows no low- T downturn (this situation is discussed briefly in Ref. 10). To retain the simplest model of the experimental configuration that gives the correct low- T behaviour, we can take $R_{el(c)} \gg R_{ph(c)}$ over the entire temperature-range considered¹¹.

Following this logic, we neglect terms of order $R_{ph(c)}/R_{el(c)}$ compared to unity in Eq. 3. We then obtain, after some rearrangement and the use of Eqs. 1 and 2 the thermal conductivity given by

$$\kappa/T = \alpha \frac{1}{1 + \frac{r}{1+r(T/T_D)^{n-1}}} + \beta T^2 \quad (5)$$

where $T_D \equiv (\alpha/Kl_s^2)^{1/(n-1)}$, l_s is the length of the sample along the direction of the current and $r \equiv R_{el(c)}/R_{el}$ will be assumed to be independent of temperature. This is a generalization of Eq. 4 valid for finite $R_{el(c)}$. At temperatures much larger than T_D the phonons and electrons in the sample are in local thermal equilibrium and Eq. 5 reduces to Eq. 2. At temperatures well below T_D the electronic contribution is limited by phonon-electron heat transfer, so that $\kappa \propto R_{el-ph}^{-1}$ due to the presence of the heat current bottleneck. Thus the electronic contribution decreases to its minimum value, $\kappa_0 = \alpha T/(1+r)$, as $\kappa - \kappa_0 \propto (T/T_D)^n$ at low temperature.

Eq. 5, along with its field-dependent generalization (Eq. 15) that applies in the vortex state, is the main result of this article. If we use the values of n and T_D that we calculate in Section III, then Eq. 5 gives a good description of the low- T data for downturns observed in the normal and superconducting states of cuprates.

III. CALCULATION OF THE PHONON-ELECTRON HEAT TRANSFER RATE IN THE NORMAL AND SUPERCONDUCTING STATE

In considering the electron-phonon thermal decoupling, we assume that the electrons in the sample remain in local thermal equilibrium at a certain temperature T_{el} . We also assume that the phonons maintain local thermal equilibrium at a temperature T_{ph} . We consider the possibility that the local electron temperature T_{el} differs from

the local phonon temperature, T_{ph} and determine the resulting rate of heat transfer from the hot phonons to the cold electrons. This transfer rate is given by¹²

$$\frac{dU}{dt} = \sum_{\mathbf{q},j} \hbar\omega_{\mathbf{q},j} \alpha_{\mathbf{qj}} c_{\mathbf{qj}} \left[n\left(\frac{\hbar\omega_{\mathbf{q},j}}{k_B T_{ph}}\right) - n\left(\frac{\hbar\omega_{\mathbf{q},j}}{k_B T_{el}}\right) \right] \quad (6)$$

where the sum is taken over phonons of wavevector \mathbf{q} , energy $\hbar\omega_{\mathbf{q},j}$, velocity $c_{\mathbf{q},j}$, and polarization j and $n(x)$ is a Bose factor. All the electronic properties are contained within the sound attenuation $\alpha_{\mathbf{qj}}$ which is calculated separately for the normal and superconducting states.

The difference between the local temperatures of phonons and electrons is at most equal to the observed temperature difference between the two ends of the sample ($T' - T_2$), which corresponds to the entire range of temperature in the system. Since this temperature range is much smaller than the average temperature for these experiments, we can expand the above expression to first order in $\Delta T = T_{ph} - T_{el}$, so that the heat transfer rate can be expressed as a electron-phonon thermal resistance

$$\frac{dU}{dt} = \Delta T / R_{el-ph}(T) \quad (7)$$

where T is the average temperature of the sample and ΔT can be thought of as the average temperature difference between the phonons and electrons over the sample volume⁹.

The fact that the thermal conductivity in PCCO is independent of field¹³ over the range from 8-13T suggests that the material is in the dirty limit, i.e. $\omega_C \tau \ll 1$ where τ^{-1} is the scattering rate due to impurities and ω_C is the cyclotron frequency. This suggestion is supported by the electrical conductivity data, which show a residual resistivity corresponding to $\hbar\tau^{-1}/k_B \approx 130\text{K}$, which gives $\omega_C \tau = 0.1$ for the highest field reported. The field has no effect on electronic transport in this limit, and we ignore it in our calculation of R_{el-ph} for this material. The condition for the low-temperature limit of the heat transfer rate: $\hbar\tau^{-1} \gg (v_f/c_s)k_B T$ (which corresponds to the low-frequency limit of sound attenuation for thermally excited phonons) is satisfied^{14,15,16,17}. With this, the well known result for the sound attenuation in metals in the low-frequency limit is appropriate for PCCO. The normal state attenuation is given by

$$\alpha_{\mathbf{qj}}^N c_{\mathbf{qj}} = 2\tau_N n_0 (\hbar\omega_{\mathbf{qj}})^2 \frac{g^2}{M c_{\mathbf{qj}}^2} < |f_j(\mathbf{k}, \mathbf{q})|^2 >_{FS} \quad (8)$$

where τ_N^{-1} is the impurity scattering rate obtained from the residual electrical resistance, n_0 is the density of states and M is the mass of a unit cell. The vector subscripts on the symbol, $\alpha_{\mathbf{qj}}^N$ for the sound attenuation should remove possible confusion with the α used in Eq. 2 to represent the electronic term in the thermal conductivity, so we will continue to use this standard

notation. The electron-phonon interaction is described as in Ref. 18, where g^2 is the coupling constant that sets the energy scale for the interaction and $f_j(\mathbf{k}, \mathbf{q})$ is the dimensionless factor which describes directional and phonon-mode dependence. The explicit form of $f_j(\mathbf{k}, \mathbf{q})$ for each high-symmetry phonon mode is given in Ref. 18.

We determine the thermal resistance using Eqs. 6, 7 and 8. Substituting Eq. 8 into Eq. 6 and using

$$n\left(\frac{\hbar\omega_{\mathbf{q},j}}{k_B T_{ph}}\right) - n\left(\frac{\hbar\omega_{\mathbf{q},j}}{k_B T_{el}}\right) = -\frac{\omega_{\mathbf{q},j}}{k_B T} \left[\frac{\partial n\left(\frac{\hbar\omega_{\mathbf{q},j}}{k_B T}\right)}{\partial \omega_{\mathbf{q},j}} \right] k_B \Delta T, \quad (9)$$

we obtain

$$\begin{aligned} \frac{dU}{dt} = k_B \Delta T (k_B T)^5 & \left[\frac{2\tau_N n_0 g^2}{\rho (2\pi\hbar)^3} \right] \\ & \sum_j \int d\Omega_{\mathbf{q}} \frac{< |f_j(\mathbf{k}, \mathbf{q})|^2 >_{FS}}{c_{\mathbf{qj}}^5} \int dx x^6 \left(-\frac{dn}{dx} \right) \end{aligned} \quad (10)$$

where ρ is the mass density and the $\Omega_{\mathbf{q}}$ integral is over all phonon directions (both $f_j(\mathbf{k}, \mathbf{q})$ and $c_{\mathbf{qj}}$ depend on the mode and direction of the phonon but not on its energy).

The electron-phonon heat transfer rate in the normal state is thus seen to be proportional to T^5 at temperatures that satisfy $\hbar\tau^{-1} \gg (v_f/c_s)k_B T$. This dependence can be understood by power counting as follows: two factors of temperature come from the $\omega_{\mathbf{q},j}^2$ dependence of the sound attenuation, three from the phonon density and an additional one from the energy of the phonon that appears in Eq. 6 with one power of temperature removed by the factor $\Delta T/T$ in Eq. 9 that describes the balance of energy exchange. We note that for the temperature T in Eqs. 9 and 10 we can use either T_{ph} or T_{el} since these expressions are already first order in ΔT .

By comparing Eq. 7 with Eq. 10 we obtain R_{el-ph}^{-1} , which is proportional to T^5 , and use this along with the definition of T_D given in the Introduction to determine the downturn temperature. The result is

$$T_D^{-4} = \frac{l_s^2}{\alpha} \left(\frac{8\pi\Gamma(7)\zeta(6)\tau_N n_0 k_B^6 g^2 \langle |f_j(\mathbf{k}, \mathbf{q})|^2 c_{\mathbf{qj}}^{-5} \rangle}{(2\pi\hbar)^3 M} \right) \quad (11)$$

where the $\langle\langle x \rangle\rangle$ indicates that the electron-phonon interaction has been averaged over the Fermi surface and phonon direction and summed over phonon modes. For the numerical estimates of T_D described in Section IV we approximate the phonon averages by using the known expressions of $f_j(\mathbf{k}, \mathbf{q})$ for high-symmetry directions and interpolating them to all other directions. The sound velocity of both transverse and longitudinal modes is taken to be isotropic. Transverse modes have a sound velocity that is significantly smaller than that for longitudinal modes, which results in a heavier weight of the contribution by the former to T_D^{-4} .

In the d-wave superconducting state, the low-temperature limit of R_{el-ph} , with its characteristic T^5

dependence is valid at temperatures such that $\gamma \gg (v_f/c_s)k_B T$, where $\hbar^{-1}\gamma$ is the zero-frequency scattering rate of nodal quasiparticles by impurities. In this limit, the sound attenuation and hence R_{el-ph} are independent of γ , which is an example of the universal behaviour of transport properties in d-wave superconductors^{19,20}. The sound attenuation in this limit is related to the normal state expression of Eq. 8 by

$$\alpha_{\mathbf{qj}}^S c_{\mathbf{qj}} = (\tau_N^{-1}/\pi v_2 k_n) \alpha_{\mathbf{qj}}^N c_{\mathbf{qj}} \frac{|f_j(\mathbf{k}_n, \mathbf{q})|^2}{|f_j(\mathbf{k}, \mathbf{q})|^2 >_{FS}} \quad (12)$$

where \mathbf{k}_n is the length of the wavevector to a node, and $\hbar v_2$ is the slope of the gap at the node. We can use Eq. 12 in Eq. 6 to obtain the T^5 dependence of the electron-phonon heat transfer rate (note that the d-wave density of states is constant at temperatures $k_B T \ll \gamma$ so that the power counting proceeds just as for the normal state). The value of T_D^{-4} for the superconducting state is obtained by multiplying the right side of Eq. 11 by the factor $\alpha_{\mathbf{qj}}^S c_{\mathbf{qj}}/\alpha_{\mathbf{qj}}^N c_{\mathbf{qj}}$ given by Eq. 12. In the resulting expression for T_D^{-4} , the matrix element $|f_j(\mathbf{k}_n, \mathbf{q})|^2$ is to be averaged over phonon direction and summed over phonon modes.

Typical experimental values of γ (which is related to τ_N by $\gamma \approx 0.61\sqrt{\Delta_0 \hbar \tau_N^{-1}}$ if we assume unitary scattering)⁷ for most clean cuprate samples suggest that the strict low- T limit of R_{el-ph} occurs below the lowest temperatures used for thermal transport measurements. We thus must determine R_{el-ph}^{-1} , which becomes weakly dependent on γ , numerically over the experimental temperature range. The result of such a computation, using realistic values of γ corresponding to between 1-15K as estimated from the position of the peak in κ as a function of temperature below T_C (see Ref. 6 and references therein), is well-described by a power law with an exponent n between 4 and 5. For example, by choosing γ to correspond to a temperature of 3K, we find an exponent of $n = 4.4$, which can be used in Eq. 5. When we compare our results to the in-field data on LSCO in Section IV, we will use the low- T (universal limit) result in order to simplify the comparison with the normal state result and the vortex state calculation described below. This result is only strictly valid within the rather narrow parameter range of $\hbar(v_f/c_s)k_B T \ll \gamma \ll \Delta_0$, but should give a reasonable description for the case of LSCO over the temperatures of interest. If we were to allow γ to vary over its experimental range, then we could adjust the value of n somewhat and perhaps improve the fit to the data. However, as seen in Section IV, the agreement between universal limit result and the available data is already very good. Thus in what follows we will use the universal limit result and will take $k_B^{-1}\gamma = 11\text{K}$ for LSCO (the zero-field result is independent of γ but γ does appear in the field-dependent calculation below) in agreement with some estimates for this material from Ref. 6. To study the magnetic field dependence of downturns

observed in the vortex state of cuprates with a field applied perpendicular to the CuO_2 planes we follow the approach of Volovik, and replace the frequency appearing in the superconducting Green's function with a Doppler-shifted value²¹. The doppler shift is given by $\hbar \mathbf{v}_s \cdot \mathbf{k}_n$ where v_s is the supercurrent velocity which depends on the position within the vortex-lattice unit cell, and \mathbf{k}_n is a wavevector directed to the d-wave node. The nodal wavevector can be used instead of the true electron momentum at the low temperatures of interest. The phonon-electron energy transfer rate can then be obtained as a position average over a simplified vortex-lattice unit cell²². Analytic results can be obtained in the clean, low- T limit defined by $\hbar \mathbf{v}_s \cdot \mathbf{k}_n \gg \gamma$, $(v_f/c_s)k_B T$, which can be expected to be valid in fields of several Tesla in many cuprates (here we refer to a typical value of $\mathbf{v}_s \cdot \mathbf{k}_n$ within the vortex-lattice unit cell). The formal similarity between the expression for the phonon attenuation rate and the thermal conductivity results in an identical field dependence for the electron-phonon heat transfer rate and the heat conductivity. The field-dependent attenuation in the superconducting state is given by

$$\alpha_{\mathbf{qj}}^S(H) = (\mathbf{v}_s \cdot \mathbf{k}_n)^2 \left(\frac{\pi}{\tau_N^{-1} v_2 k_n} \right) \alpha_{\mathbf{qj}}^S(0) \quad (13)$$

where v_2 is the magnitude of the slope of the superconducting gap at the node and $\alpha_{\mathbf{qj}}^S(0)$ is given by Eq. 12. The expression for the superconducting impurity scattering rate in the clean, unitary limit $\gamma = \tau_N^{-1} \Delta_0 / 2 \mathbf{v}_s \cdot \mathbf{k}_n$ has been used. The electronic thermal conductivity is given in the same limit by

$$\frac{\kappa_{el}(H)}{T} = (\mathbf{v}_s \cdot \mathbf{k}_n)^2 \left(\frac{\pi}{\tau_N^{-1} v_2 k_n} \right) \frac{\kappa_{el}(0)}{T} \quad (14)$$

Eq.'s 13 and 14 are to be averaged over a unit cell of the vortex lattice. We follow Ref. 22 in performing the vortex unit cell average appropriate for fields applied perpendicular to the planes and assume unitary scattering. Both the thermal conductivity and the sound attenuation (and consequently R_{el-ph}^{-1}) are thus found to be proportional to $\sqrt{H/H_{C2}}$. If we add the zero field universal result we have a decent description of the field dependences of $\alpha(H)$ and R_{el-ph}^{-1} for applied fields up to some fraction of H_{C2} . We then substitute into Eq. 5 to obtain

$$\kappa_{el}/T = \frac{\eta(H)\alpha}{1 + \frac{\eta(H)r}{1+\eta(H)r(T/T_D)^{n-1}}} \quad (15)$$

with

$$\eta(H) = 1 + \sqrt{\frac{H}{H_{C2}}} \frac{\alpha'}{\alpha} \quad (16)$$

where $\alpha' = a(3\pi/8)\sqrt{\Delta_0 \hbar^{-1} \tau_N} \alpha$, and a is a factor of order unity that arises because we roughly approximated the vortex lattice geometry. The field dependence of the expression is contained entirely in $\eta(H)$. The factors r and T_D are to be evaluated at $H = 0$.

IV. COMPARISON WITH THE DATA

In Fig. 2 and Fig. 4 we show data for the field-induced normal state of optimally doped PCCO¹ and for superconducting, optimally doped LSCO⁵, respectively. The plots are of the extracted electronic contribution to κ/T versus T . Thus the phonon contribution has been subtracted from the data to leave only that of the electrons.

According to our discussion in Section II, the phonon contribution could be identified by fitting the data at temperatures well above T_D to Eq. 5. In practice, the second term in Eq. 5 often does not adequately describe the phonon contribution (for example, slightly weaker temperature power-laws are often needed as discussed in Ref. 6). For our purposes, as long as the phonon contribution is determined from data at temperatures well above T_D , then once it has been subtracted we can use our results above and consider only the first term in Eq. 5. In Ref. 1, the authors describe in detail how the phonon subtraction was performed for the PCCO data. For the LSCO data, we performed the phonon subtraction by first fitting the data of Ref. 5 at temperatures above the downturn (and below the maximum temperature shown on Fig. 4) to the form $\kappa/T = \alpha + \beta T^m$ where m was between 1 and 2 and then subtracting the βT^m term. For both the LSCO data and the PCCO data, the electron contribution to κ/T is seen to be independent of temperature for $T \gg T_D$.

We compare the result of Eq. 5 in the normal state to the PCCO data in Fig. 2. For the result shown we used the parameters: $\alpha = 1.75 \text{ mWK}^{-2} \text{ cm}^{-1}$ that we take from the data at temperatures above the downturn, $n = 5$ that we calculated in Section III, $T_D = 160 \text{ mK}$, which was chosen to fit the data and $r = 29$ that was chosen to fit the data. Evidently Eq. 5 captures the low- T behaviour of the data. The large value of r indicates that heat is carried into the sample almost exclusively by phonons. Indeed, taking $r = \infty$ gives a reasonably good description of the PCCO data. This can be seen from Fig. 3, in which we show the result of Eq. 5 for various values of r .

In order to determine whether the value of $T_D = 160 \text{ mK}$ is realistic for this material, we can compare this value with an estimate based on Eq. 11. All the parameters in Eq. 11 are known for cuprates except the electron-phonon coupling constant, g^2 . The value of g^2 has been obtained from sound attenuation measurements²³ on Sr_2RuO_4 , another material to which the given form of the electron-phonon interaction is applicable, and found as $g^2 = (12 \text{ eV})^2$. Although the electron-phonon coupling constant may vary significantly for different materials, we can expect order-of-magnitude agreement between the value of g^2 for Sr_2RuO_4 and that for PCCO. Using Eq. 11 and the value $T_D = 160 \text{ mK}$, we calculate the electron phonon coupling constant for PCCO and find $g^2 = (6 \text{ eV})^2$. Thus, the coupling constant for PCCO determined using the fit above roughly agrees with the value measured for Sr_2RuO_4 .

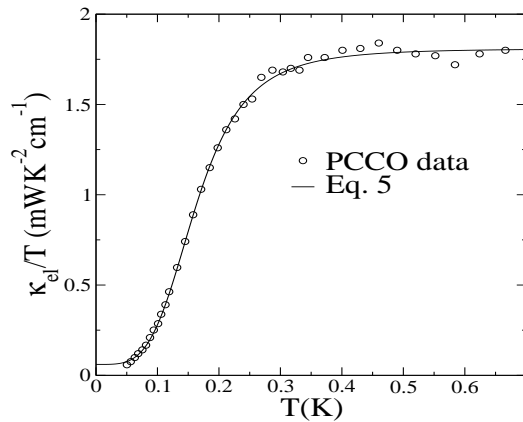


FIG. 2: The electronic thermal conductivity κ_{el}/T measured¹ for normal state $\text{Pr}_{2-x}\text{Ce}_x\text{CuO}_{7-\delta}$ is plotted versus T along with

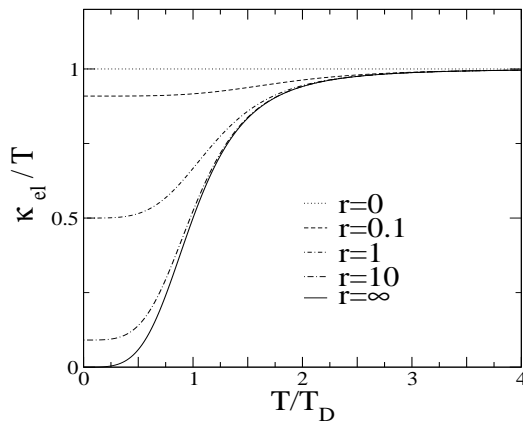


FIG. 3: κ_{el}/T , normalized to its high- T value, determined by Eq. 5 is plotted versus T for various values of the “contact” resistance r .

In Fig. 4 the result of Eq. 5 for the case of the d-wave superconductor in the low-temperature limit as a function of applied magnetic field is compared to the LSCO data. Here, the zero-field result was obtained using the values: $\alpha = 0.18 \text{ mWK}^{-2} \text{ cm}^{-1}$, $n = 5$, $T_D = 80 \text{ mK}$, $r = 0.86$ that were chosen exactly as for the PCCO case described above. In order to obtain the 13T result, we use Eq. 15 with the values of α , n , T_D , and r fixed by the zero-field fit (the two curves in Fig. 4 were measured for the same sample). The remaining unknown factor of order unity, a , that appears in Eq. 15 is obtained from the 13T data at temperatures well above the downturn (we thus find $a = 0.5$). With this done, the low-temperature downturn for the 13T curve is determined without any adjustable parameters. The clear agreement with the data thus provides strong support for our model.

To check whether the value of $T_D = 80 \text{ mK}$ for superconducting LSCO is reasonable we compare it with the value determined in Section III. We note that the electronic thermal conductivity in the universal limit

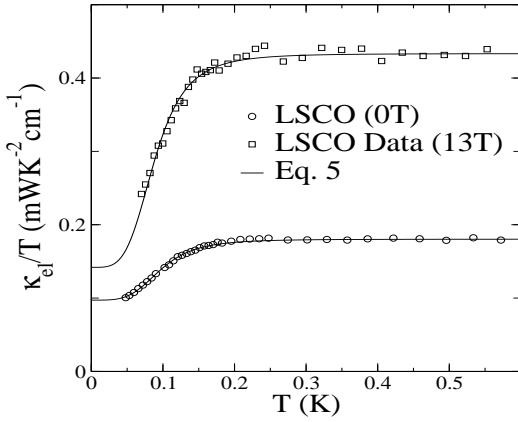


FIG. 4: The electronic thermal conductivity κ_{el}/T versus T measured⁵ for $\text{La}_{2-x}\text{Sr}_x\text{CuO}_4$ in the superconducting state in zero field and in a field of 13T applied perpendicular to the CuO_2

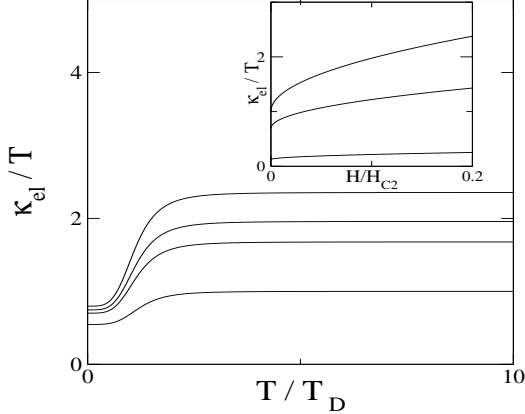


FIG. 5: Main Panel: κ_{el}/T , normalized to its high- T zero-field value, versus T in varying magnetic fields as given by Eq. 15. From the bottom, the curves are for fields of 0, 0.05, 0.1, and 0.2 H_{C2} applied perpendicular to the CuO_2 planes. Inset: The same quantity plotted versus H at various temperatures. From the bottom, the curves are for temperatures of 0.1, 1, and 10 T_D .

is related to the normal state Boltzmann expression by an expression similar to Eq. 12, namely $\kappa_0^S/T = \kappa_0^N/T(\tau_N^{-1}/\pi v_2 k_n)$. As a result, the value of T_D determined using the low- T results for both R_{el-ph} and $\alpha = \kappa_0/T$ is roughly the same in the superconducting state of cuprates as it was for the normal state. From Eq. 11 (also, recalling the discussion immediately following Eq. 12) we find that the value of $T_D = 80\text{mK}$ corresponds to $g^2 = (7\text{eV})^2$. The agreement of the electron-phonon coupling constants for PCCO and LSCO is expected since g^2 is a normal state property of the CuO_2 planes.

The parameter $r \equiv R_{el(c)}/R_{el}$ determines the extent of the low- T downturn, i.e. the value of κ/T at $T = 0$. The value of r corresponds to the thermal resistance to heat being transmitted to the electrons in the sample

by any means other than via phonon-electron heating within the sample. We have not attempted to describe this complicated process and have treated r as an unknown parameter. However, if we assume that the main contribution to r comes from the resistance of the current contacts, which is usually measured during thermal conductivity experiments, then we can compare the values of r obtained above to the measured values. We convert the thermal conductivity of the sample measured well above the downturn to an electrical resistance by using the Wiedemann Franz law, according to which

$$R_{el} = \frac{l_s}{A} L_0 \left(\frac{\kappa_{el}}{T} \right)^{-1} \quad (17)$$

where A is the cross-sectional area of the sample and L_0 is the Lorenz number. For the PCCO measurements, the contacts were reported¹ to have a measured resistance of roughly 1 Ω . So, using the reported sample dimensions, $l_s = 1\text{ mm}$, $A = 38\text{ }\mu\text{m} \times 720\text{ }\mu\text{m}$ and using $R_{el(c)} = 2\Omega$ (the factor of two mentioned in the discussion following Fig. 1 has been included) we get $r = 240$. This is substantially larger than the best-fit value of 29, but from Fig. 3 is seen to still be reasonably consistent with the data. For the LSCO data, the reported contact resistance was roughly 10 m Ω . The dimensions of the sample²⁴ for which the data above was obtained were $l_s = 0.97\text{ mm}$, $A = 1.26\text{ mm} \times 0.212\text{ mm}$. From this we obtain $r = 1.8$, in rough agreement with the value $r = 0.9$ used above. We do not expect precise agreement between the r obtained above and the measured contact resistance, but it is clear that the values of r used in our fits are not out of line with the measured values.

The data shown above was chosen for being representative of that in which downturns are seen. Other examples of downturns, some of which were discussed in Section I, typically resemble those shown above and occur at roughly the same temperature.

V. DISCUSSION

We have shown that the low- T downturn in the electronic contribution to thermal conductivity, for PCCO in the normal state and LSCO in the superconducting state, results from the loss of thermal equilibrium between phonons and electrons caused by poor heat transfer between these systems at low-temperature. The electron-phonon heat transfer rate has been shown by calculation to have both a characteristic temperature dependence in the normal state, and a characteristic temperature dependence and magnetic field dependence in the superconducting state. The fact that these dependences are in excellent agreement with experimental observations in PCCO and LSCO and provides a striking confirmation that poor electron-phonon heat transfer at low temperatures is indeed the right mechanism to explain the downturns in these materials. Clearly this is a mechanism

that should also be considered in connection with thermal conductivity downturns in other materials²⁵²⁶. If one seeks to measure the electronic contribution to κ/T at zero temperature, the role of electron-phonon thermal decoupling has to be carefully considered. Alternatively, one could study the downturns to extract information about the electron-phonon interactions, without regard to the underlying thermal conductivity.

Thermal conductivity data at temperatures below T_D that shows a significant downturn could be used to study electron-phonon interaction effects in unconventional superconductors and other systems of interest. In d-wave superconductors, the electron-phonon heat transfer rate calculated within the dirty d-wave approach has many of the same features as the thermal conductivity. We showed that it has the same magnetic field dependence at low-temperature, and also shows universal behaviour (albeit at a temperature which is smaller by a factor of the ratio of the speed of sound to the fermi velocity, c_s/v_f than that at which universality occurs in the thermal conductivity.¹⁹²⁰²⁷) At higher temperatures, the field and temperature dependences of the heat transfer rate have different forms from those of thermal conduc-

tivity, and provide complementary information about the electrons. The basic form of the electron-phonon matrix elements, which affects the heat transfer rate, also has unusual properties in layered square-lattice tight-binding materials¹⁸²³.

VI. ACKNOWLEDGMENTS

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† Present address: National Synchrotron Research Center, Nakhon Ratchasima, Thailand. †† Present address: Department of Physics, University of California, San Diego, La Jolla, CA, USA. ‡ Members of the Quantum Materials Program of the Canadian Institute for Advanced Research.

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 - ⁹ We have evidently treated the geometry very crudely, and in a more accurate treatment would be concerned with the relative positions of the current contacts and the thermometers on the sample and the effective sample volume over which phonon-electron heat transfer is relevant. For example, in Eq. 5 the factor l_s^2 contains one factor of length coming from R_{el} that should be equal to the distance between the thermometer contacts and one factor of length coming from R_{el-ph} that should be equal to the effective length over which phonon-electron heat transfer is significant. We ignore these details and use a single length parameter l_s . Since all the relevant distances (i.e. between contacts, from contact to sample ends etc.) are typically reasonable fractions of the sample length, we expect that

- our result will be correct to within a factor of unity. The value of the electron-phonon coupling constant g^2 is thus determined to within such a factor.
- ¹⁰ For example, in the opposite limiting case, when $f(T_3, T_4) = T_3$, the contribution of the phonons to the thermal current is reduced by an energy bottleneck at low- T . There will be a low- T upturn of κ/T occurring at $T_U = (\beta/Kl_s^2)^{\frac{1}{\pi-3}}$.
- ¹¹ The same result, Eq. 5, can be obtained by taking the combined limits $R_{ph(c)} \ll \max(R_{el(c)}, R_{el-ph})$ and $R_{ph(c)} \ll R_{ph}$.
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- ²⁵ In measurements made on superconducting $\text{YBa}_2\text{Cu}_4\text{O}_8$ (Ref.26), the thermal conductivity data showed no downturn with decreasing temperature, but the extracted T -linear term in the conductivity was negligible. This is likely another example of electron-phonon thermal-decoupling.
- The low- T electronic heat current may be reduced, but the T -dependent downturn obscured by the subtraction of the phonon contribution.
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